Matrices and Determinants

Date Planned : __ / __ / __	Daily Tutorial Sheet – 13	Expected Duration : 90 Min
Actual Date of Attempt : __ / __ / __	Level – 3	Exact Duration :________

156. Solve for x, $\begin{vmatrix} x^2-a^2 & a^2-b^2 & x^2-c^2 \\ (x-a)^3 & (x-b)^3 & (x-c)^3 \\ (x+a)^3 & (x+b)^3 & (x+c)^2 \end{vmatrix} = 0, a \neq b \neq c$.

157. Prove that $\Delta = \begin{vmatrix} a & c & c-a & a+c \\ c & b & b-c & b+c \\ a-b & b-c & 0 & a-c \\ x & y & z & 1+x+y \end{vmatrix} = 0$

Implies that a,b,c are in A.P. or a,c,b are in G.P.

158. If $f(x)$ is a polynomial of degree < 3, prove that $\begin{vmatrix} 1 & a & f(a)/(x-a) \\ 1 & b & f(b)/(x-b) \\ 1 & c & f(c)/(x-c) \end{vmatrix} \div \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = \dfrac{f(x)}{(x-a)(x-b)(x-c)}$

159. Prove that for any A.P. $a_1, a_2, a_3, \ldots$ the determinant

$$\begin{vmatrix} a_p + a_{p+m} + a_{p+2m} & 2a_p + 3a_{p+m} + 4a_{p+2m} & 4a_p + 9a_{p+m} + 16a_{p+2m} \\ a_q + a_{q+m} + a_{q+2m} & 2a_q + 3a_{q+m} + 4a_{q+2m} & 4a_q + 9a_{q+m} + 16a_{q+2m} \\ a_r + a_{r+m} + a_{r+2m} & 2a_r + 3a_{r+m} + 4a_{r+2m} & 4a_r + 9a_{r+m} + 16a_{r+2m} \end{vmatrix} = 0$$

160. Let n and r be two positive integers such that $n \geq r + 2$ and $\Delta(n,r) = \begin{vmatrix} {}^nC_r & {}^nC_{r+1} & {}^nC_{r+2} \\ {}^{n+1}C_r & {}^{n+1}C_{r+1} & {}^{n+1}C_{r+2} \\ {}^{n+2}C_r & {}^{n+2}C_{r+1} & {}^{n+2}C_{r+2} \end{vmatrix}$. Show that $\Delta(n,r) = \dfrac{{}^{n+2}C_3}{{}^{r+2}C_3}\Delta(n-1, r-1)$. Hence or otherwise, prove that $\Delta(n,r) = \dfrac{{}^{n+2}C_3\ {}^{n+1}C_3 \ldots {}^{n-r+3}C_3}{{}^{r+2}C_3\ {}^{r+1}C_3 \ldots {}^{3}C_3}$.

161. Show that in general there are three Values of t for which the following system of equations has a nontrivial solution:

$$(a-t)x + by + cz = 0$$
$$bx + (c-t)y + az = 0$$
$$cx + ay + (b-t)z = 0$$

Express the product of these values of t in the form of a determinant.